ARKIV FÖR FYSIK Band 13 nr 28

Communicated 8 January 1958 by MANNE SIEGBAHN and IVAR WALLER

The cosmic ray photon and π^0-meson energy spectra at 29–30 km above sea-level

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With 12 figures in the text

SUMMARY

The photon spectrum at 29–30 km above sea-level and 64°–65° geomagnetic latitude has been determined by means of the nuclear emulsion technique, the multiple scattering of the electron-pairs, produced when the photons materialize in the emulsion being measured. Distortion, noise, energy loss of the electrons, outscattering and materialization probability have been taken into account. The relation between the registered photon spectrum and the π^0-meson spectrum has been studied. The exponent obtained for the differential energy spectrum of the π^0-mesons is 1.5 ± 0.2 in the total energy interval from 0.25 GeV to 3 GeV and 2.8 ± 0.4 for energies greater than 3 GeV. The last exponent may suffer from systematic errors due to distortion, spurious scattering and scanning efficiency, which are difficult to estimate.

A. Some theoretical relations between the π^0-meson and the photon spectra

I. Introduction

If photographic emulsions are exposed at high altitude to cosmic rays, there will appear in them after development a great number of electron-pairs, which are almost certainly created by the photons resulting from the decay of the π^0-mesons, either directly or through an intervening electron-photon cascade. The relation between the π^0-meson production energy spectrum and the photon spectrum resulting from the decay of the π^0-mesons has been studied by Carlson *et al.* (1), Oehme (2) and Tiapkin (3) among others. In this part of the present work we will deduce a rigorous mathematical relation which makes it possible to derive the energy spectrum of the π^0-mesons from the distribution in zenith angle and energy of the pairs observed in a certain volume of the photographic emulsion.

II. The general relation

We start with the deduction of the photon spectrum $N(E, \theta_\gamma)$ at the point Q in fig. 1 from the π^0-meson production spectra $S(\varepsilon, \theta, r, \theta_\gamma)$ at the points $P=(r, \theta_\gamma\, \varphi_\gamma)$. $N(E, \theta_\gamma) dE\, d\omega_\gamma$ is the number of photons per second and cm^2 with the energy

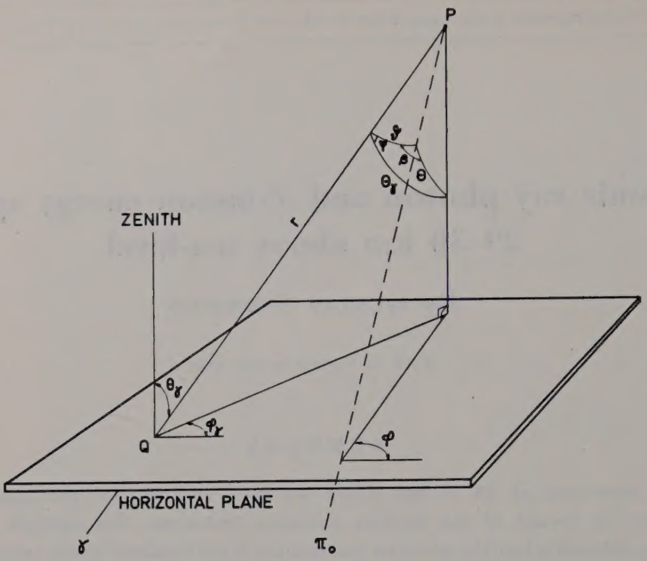

Fig. 1. Notations of the angles used when considering the relation between the photon-spectrum at Q and the π^0-meson production spectrum at P.

in $(E, E + dE)$ and the zenith angle within the solid angle $d\omega_\gamma$ around θ_γ and $S(\varepsilon, \theta, r, \theta_\gamma)\, dV\, d\varepsilon\, d\omega$ is the number of π^0-mesons produced per second in the volume dV at the point P with the total energy in $(\varepsilon, \varepsilon + d\varepsilon)$ and the zenith angle within $d\omega$ around θ. Let $H(E', r, \theta_\gamma)\, dV\, dE'\, d\omega$ be the number of photons produced by decaying π^0-mesons per second in the volume dV at the point P with the energy in $(E', E' + dE')$ and the direction towards Q within the solid angle $d\omega$. The relation between H and S has been published without derivation by Tiapkin (3). Because its validity does not seem to be self-evident, a derivation is outlined here, the notations introduced in Fig. 1 being used. A photon with the energy E' can be created by a meson with the energy ε if the angle ϑ between their directions satisfies the relation

$$\cos\vartheta = \frac{\varepsilon - \varepsilon_0^2/2E'}{\sqrt{\varepsilon^2 - \varepsilon_0^2}} \tag{1}$$

where ε_0 is the rest energy of the π^0-meson. This relation is easily obtained, if one considers that the two decay photons have the same energy $\frac{1}{2}\varepsilon_0$ and opposite directions in the center of mass system. The probability $f(E' \mid \varepsilon)\, dE'$ that a meson with the energy ε decays with one photon having its energy in the interval $(E', E' + dE')$ has the simple form

$$f(E' \mid \varepsilon) = \begin{cases} 2/\sqrt{\varepsilon^2 - \varepsilon_0^2} & \text{if } \frac{1}{2}(\varepsilon - \sqrt{\varepsilon^2 - \varepsilon_0^2}) \leq E' \leq \frac{1}{2}(\varepsilon + \sqrt{\varepsilon^2 - \varepsilon_0^2}) \\ 0 & \text{in other cases.} \end{cases} \tag{2}$$

The probability that one photon will appear in the interval $(\frac{1}{2}(\varepsilon - \sqrt{\varepsilon^2 - \varepsilon_0^2}), \frac{1}{2}\varepsilon)$ is one. A photon in this energy interval is always accompanied by a photon in the interval $(\frac{1}{2}\varepsilon, \frac{1}{2}(\varepsilon+\sqrt{\varepsilon^2-\varepsilon_0^2}))$. From this probability distribution follows that a photon with the energy E' can result from the decay of a meson with an energy greater than

$$\varepsilon(E') = E' + \frac{\varepsilon_0^2}{4E'} \tag{3}$$

and only from such a meson. The relations (2) and (3) are derived in ref. (1). Let $f(E', \theta_\gamma | \varepsilon, \theta, \varphi)\, dE'\, d\theta_\gamma$ be the probability that one of the photons, emitted in the decay of a π^0-meson with the total energy ε and the direction (θ, φ), will have its energy E' within $(E', E'+dE')$ and its zenith angle within $(\theta_\gamma, \theta_\gamma + d\theta_\gamma)$. Because the distance travelled by a π^0-meson before decay is negligible, the relation between $H(E', r, \theta_\gamma)$ and $S(\varepsilon, \theta, r, \theta_\gamma)$ is

$$H(E', r, \theta_\gamma)\, dV\, dE'\, 2\pi \sin\theta_\gamma\, d\theta_\gamma = \int_{\varepsilon,\theta,\varphi} S(\varepsilon, \theta, r, \theta_\gamma)\, dV\, d\varepsilon\, d\omega\, f(E', \theta_\gamma | \varepsilon, \theta, \varphi)\, dE'\, d\theta_\gamma$$

where the integration should be performed over those ε, θ and φ which can contribute. The expression can be simplified to

$$H(E', r, \theta_\gamma) = \frac{1}{2\pi\sin\theta_\gamma} \int_{\varepsilon,\theta,\varphi} S(\varepsilon, \theta, r, \theta_\gamma)\, f(E', \theta_\gamma | \varepsilon, \theta, \varphi)\, d\varepsilon\, d\omega. \tag{4}$$

We can write

$$f(E', \theta_\gamma | \varepsilon, \theta, \varphi) = f(E' | \varepsilon)\, f(\theta_\gamma | E', \varepsilon, \theta) \tag{5}$$

where $f(E' | \varepsilon)$ has been defined above (2) and where $f(\theta_\gamma | E', \varepsilon, \theta)\, d\theta_\gamma$ is the probability that a photon with the energy E', which is emitted from a decaying π^0-meson with the energy ε and the zenith angle θ, has its zenith angle within $(\theta_\gamma, \theta_\gamma + d\theta_\gamma)$. For E', ε and θ given θ_γ is determined from the relation

$$\cos\theta_\gamma = \cos\vartheta\cos\theta + \sin\vartheta\sin\theta\cos\beta \tag{6}$$

where ϑ is a function of E' and ε given by the relation (1). The probability $g(\beta | E', \varepsilon, \theta)\, d\beta$ that β will take a value within $(\beta, \beta + d\beta)$ for fixed values of E', ε and θ is $d\beta/\pi$. Because under these conditions θ_γ is a function of β only, we have

$$f(\theta_\gamma | E', \varepsilon, \theta)\, d\theta_\gamma = g(\beta | E', \varepsilon, \theta)\, d\beta$$

which yields

$$f(\theta_\gamma | E', \varepsilon, \theta) = \frac{1}{\pi}\frac{d\beta}{d\theta_\gamma}$$

or after differentiation of (6) and elimination of $d\beta/d\theta_\gamma$:

$$f(\theta_\gamma | E', \varepsilon, \theta) = \frac{\sin\theta_\gamma}{\pi\sin\vartheta\sin\theta\sin\beta}. \tag{7}$$

From (2), (3), (4), (5) and (7) we obtain

$$H(E', r, \theta_\gamma) = \frac{1}{\pi^2} \int\limits_{\varepsilon(E')}^{\infty} \frac{d\varepsilon}{\sqrt{\varepsilon^2 - \varepsilon_0^2}} \int\limits_{\theta,\varphi} \frac{S(\varepsilon, \theta, r, \theta_\gamma)\, d\omega}{\sin\vartheta \sin\theta \sin\beta}.$$

Because S is assumed to be independent of φ, this angle can be integrated out, and putting $d\omega = \sin\theta\, d\theta\, d\varphi$ we obtain

$$H(E', r, \theta_\gamma) = \frac{2}{\pi} \int\limits_{\varepsilon(E')}^{\infty} \frac{d\varepsilon}{\sqrt{\varepsilon^2 - \varepsilon_0^2}} \int\limits_{\theta} \frac{S\, d\theta}{\sin\vartheta \sin\beta}. \tag{8}$$

There now remains a change of variables. We want to replace β by ψ and integrate over the latter variable instead of over θ. For this purpose we can use the spherical trigonometric relation

$$\cos\theta = \cos\vartheta \cos\theta_\gamma + \sin\vartheta \sin\theta_\gamma \cos\psi \tag{8a}$$

which after differentiation gives

$$\sin\theta\, d\theta = \sin\vartheta \sin\theta_\gamma \sin\psi\, d\psi.$$

If we make use of another relation from spherical trigonometry,

$$\sin\theta \sin\beta = \sin\theta_\gamma \sin\psi$$

we obtain

$$d\psi = \frac{d\theta}{\sin\vartheta \sin\beta}.$$

When this is substituted into (8), we obtain the integration limit upon geometrical consideration of fig. 1 and

$$H(E', r, \theta_\gamma) = \frac{2}{\pi} \int\limits_{\varepsilon(E')}^{\infty} \frac{d\varepsilon}{\sqrt{\varepsilon^2 - \varepsilon_0^2}} \int\limits_{0}^{\pi} S(\varepsilon, \theta, r, \theta_\gamma)\, d\psi \tag{9}$$

which is the final expression for the relation between H and S. In order to get $N(E, \theta_\gamma)$, let us consider the photons that pass through the area dS in fig. 2 with the energy in $(E, E + dE)$ and the direction within the solid angle $d\omega_\gamma$ around $(\theta_\gamma, \varphi_\gamma)$ and originate from decay photons in the distance-interval $(r, r + dr)$. If the lateral spread of the shower is left out of consideration, the later photons must have been created in the volume

$$dV = r^2\, d\omega_\gamma\, dr$$

and the number per second of the former photons is

$$dN(E, \theta_\gamma)\, dE\, d\omega_\gamma\, dS = dE \int\limits_{E'=E}^{\infty} dE'\, \gamma(E, E', r, \theta_\gamma)\, H(E', r, \theta_\gamma)\, dV\, d\omega$$

where $d\omega = dS/r^2$ and where $\gamma(E, E', r, \theta_\gamma)\, dE$ is the average number of photons with the energy in the interval $(E, E + dE)$ that arrive at Q as a result of a photon

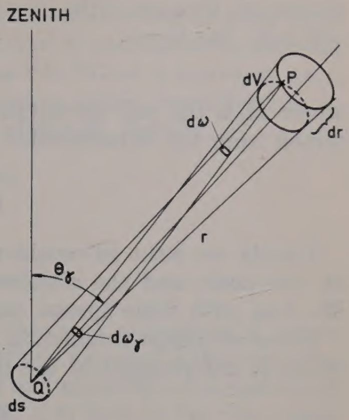

Fig. 2. The solid angles occurring in the deduction of the relation between the photon-spectrum at Q and the π^0-meson production spectrum in the vicinity.

with the energy E' starting at P moving towards Q. γ can be obtained from shower theory. The above expression may be simplified to

$$d\,N(E,\theta_\gamma)=d\,r\int_E^\infty d\,E'\,\gamma\,(E,E',r,\theta_\gamma)\,H\,(E',r,\theta_\gamma)$$

and from this we obtain

$$N\,(E,\theta_\gamma)=\int_0^\infty d\,r\int_E^\infty d\,E'\,\gamma\,(E,E',r,\theta_\gamma)\,H\,(E',r,\theta_\gamma) \tag{10}$$

which is the final expression for the relation between N and H.

The photons are detected by inspection of the pairs that they produce in a photographic plate. The finite extension of the detector may be taken into account in the following way. Let $N(Q,E,\theta_\gamma)$ be the photon spectrum at the point Q of the detector and let $P(Q,E,\theta_\gamma)\,d\,V\,d\,E\,d\omega_\gamma$ be the number of pairs that are created per unit time in the volume $d\,V$ around Q with the energy in $(E, E+d\,E)$ and the zenith angle within the solid angle $d\,\omega_\gamma$ around θ_γ. The direction of a pair is supposed to be the same as that of the parent photon. Let $\sigma(E)$ be the cross-section per atom for pair-production and n the number of atoms per cm^3. We have

$$P(Q,E,\theta_\gamma)=n\,\sigma N(Q,E,\theta_\gamma). \tag{11}$$

The number of pairs per unit energy and solid angle created in the volume V during the time T is

$$R\,(E,\theta_\gamma)=\int_{V,\,T} P\,(Q,E,\theta_\gamma)\,d\,V\,d\,t \tag{12}$$

In principle this relation between the pair spectrum in a volume and the π^0-meson production spectrum can be used when deducing the latter from the former. For instance one can assume

$$S=a_1S_1+a_2S_2+\cdots$$

where S_1, S_2, ... are suitably chosen meson production spectra and a_1, a_2, ... are constants. Because of the linearity of the relations (12), (10) and (9) the pair spectrum will take the form

$$R = a_1 R_1 + a_2 R_2 + \cdots$$

where R_i is the pair spectrum corresponding to S_i. The coefficients a_i can be determined from the experimental pair spectrum.

III. Some special cases

Finally we want to consider the relation between the photon spectrum $N(E, \theta_\gamma)$ at one point and the π^0-meson production spectrum $S(\varepsilon, \theta, r, \theta_\gamma)$ in the vicinity. We deal with some special cases, neglecting cascade processes.

If the π^0-meson production spectrum is isotropic and has an energy dependence, which is independent of the position of the point $P = (r, \theta_\gamma, \varphi_\gamma)$, we can put

$$S(\varepsilon, \theta, r, \theta_\gamma) = f(r, \theta_\gamma)\, F(\varepsilon).$$

Because S is now independent of ψ for fixed values of ε, E and θ_γ (10) and (9) take the form

$$N(E, \theta_\gamma) = k(\theta_\gamma) \int\limits_{\varepsilon(E)}^{\infty} \frac{F(\varepsilon)\, d\varepsilon}{\sqrt{\varepsilon^2 - \varepsilon_0^2}} \tag{13}$$

where
$$k(\theta_\gamma) = 2 \int\limits_0^{\infty} f(r, \theta_\gamma)\, dr.$$

From this the well known relationship

$$F\left(E + \frac{\varepsilon_0^2}{4E}\right) = \frac{1}{k(\theta_\gamma)}\, E \left|\frac{dN}{dE}\right| \tag{14}$$

can be deduced [ref. (1)].

The following simple model seems to be useful in the study of the cosmic radiation in the upper part of the atmosphere:

$$S = \begin{cases} f(r, \theta_\gamma)\, F(\varepsilon) & \text{if } \theta < \tfrac{1}{2}\pi \\ 0 & \text{if } \theta > \tfrac{1}{2}\pi. \end{cases}$$

If we put
$$I(\varepsilon, r, \theta_\gamma) = \int\limits_0^{\pi} S(\varepsilon, \theta, r, \theta_\gamma)\, d\psi,$$

we obtain:

$$I = F \cdot f \cdot \left\{ \begin{array}{llll} \pi & \text{if } 0 < \theta_\gamma < \tfrac{1}{2}\pi, & 0 < \vartheta < \tfrac{1}{2}\pi - \theta_\gamma \\ \psi_0(\vartheta, \theta_\gamma) & ,, & ,, & \tfrac{1}{2}\pi - \theta_\gamma < \vartheta < \tfrac{1}{2}\pi + \theta_\gamma \\ 0 & ,, & ,, & \tfrac{1}{2}\pi + \theta_\gamma < \vartheta < \pi \\ 0 & ,, \ \tfrac{1}{2}\pi < \theta_\gamma < \pi, & 0 < \vartheta < \theta_\gamma - \tfrac{1}{2}\pi \\ \psi_0(\vartheta, \theta_\gamma) & ,, & ,, & \theta_\gamma - \tfrac{1}{2}\pi < \vartheta < 3\pi/2 - \theta_\gamma \\ \pi & ,, & ,, & 3\pi/2 - \theta_\gamma < \vartheta < \pi \end{array} \right.$$

where $\psi_0(\vartheta, \theta_\gamma) = \arccos(-\cot\theta_\gamma \cot\vartheta)$.

In order to obtain these results we have to find out, for what values of ψ we have $0 < \theta < \frac{1}{2}\pi$. This depends on θ_γ and ϑ, where ϑ is a function of E' and ε given by formula (1). From fig. 1 we see that $\theta = \theta_{\min} = |\theta_\gamma - \vartheta|$ when $\psi = 0$. When ψ increases to π, θ increases to $\theta_{\max} = \theta_\gamma + \vartheta$ or $2\pi - (\theta_\gamma + \vartheta)$, the expression, which is smaller than π, being correct. From (8a) the condition for $\theta = \frac{1}{2}\pi$ is obtained, namely

$$\cos\vartheta \cos\theta_\gamma + \sin\vartheta \sin\theta_\gamma \cos\psi = 0,$$

which can be written as

$$\psi = \psi_0(\vartheta, \theta_\gamma) = \arccos\,(-\cot\vartheta \cot\theta_\gamma).$$

When $|\cot\vartheta \cot\theta_\gamma| > 1$, ψ_0 does not exist and all values of ψ between 0 and π can contribute. If on the other hand $\theta_{\min} > \frac{1}{2}\pi$, there are no values of ψ that can contribute. The result written down above is obtained as an immediate consequence of these two conditions. It should be noted that (13) and (14) hold in the two cases, when $I = F f \pi$.

B. An experimental investigation of the photon and π^0-meson spectra at high altitude

I. Introduction

The main properties of the cosmic radiation are fairly well known nowadays. It is most likely that the primary cosmic radiation consists of protons and heavier nuclei. The nucleons have a differential energy spectrum $N(E)dE$. The exponent

$$p = -\frac{\partial \ln N}{\partial \ln E}$$

of this distribution is a slowly increasing function of the energy E and seems to tend to a constant value for great energies E. Experiments with extensive air showers (4) have indicated that the variation of p is very small above 14 GeV, the average value between 14 and $1.4 \cdot 10^6$ GeV being 2.64 ± 0.04. If, for a rough estimate, we assume the same exponent 2.6 between 1 GeV and infinity, we find that in this energy region, where multiple meson production is important, less than 10 % of the total energy is carried by primary nucleons with an energy greater than 50 GeV. Only at energies greater than this the total production cross-section for nucleon-antinucleon-pairs, hyperons and K-mesons is believed to be of the same order of magnitude as the production cross-section for π-mesons (5). The available experimental determinations of the ratio of the cross-sections involve many assumptions. In an analysis of jets, caused by nuclear collisions of energy between 10^3 and 10^4 GeV, Brisbout *et al.* have found that only about 25 % of the shower particles are not π-mesons (6). It can be concluded that heavy mesons, hyperons and antinucleons play no major part in the development of the local cosmic radiation. It is also well established that high energy electrons or photons give no large contribution to the primary cosmic radiation. As has been pointed out by Rossi (7) an experiment by Hulsizer shows that not more than about one percent of the primary particles with energies above 4.5 GeV are electrons or photons. The electron-photon component in the local cosmic radiation is instead considered to come mainly from the decay of π^0-mesons, which in high energy nuclear reactions, where multiple meson production is of importance, have been found to contribute $^1/_3$ of the total number of produced π-mesons (1, 8). There are other established reactions, which can give rise to high energy photons or electrons, for instance:

1) $\Sigma^0 \rightarrow \Lambda^0 + \gamma$
2) $\mu \rightarrow \beta + \nu + \nu$
3) Inner Bremsstrahlung (Abstrahlung)
4) Bremsstrahlung

The first three of these reactions are certainly not very important at high altitude. The cross-section for the creation of Σ^0 is certainly much smaller than the cross-section for the production of π^0-mesons in the most important energy region. The reaction 2) is certainly not important at high altitude, among other things because of the comparatively long lifetime of the μ-meson. The energy contribution to the photon-component from inner Bremsstrahlung, emitted when the charged π-mesons are created is less than 2 % of the contribution from the decay of π^0-mesons (2, 9). Bremsstrahlung from particles heavier than electrons is negligible. Bremsstrahlung from electrons cannot be ruled out at once, being an important part of the electron-photon cascade process, the possible contribution of which will be discussed in section III.

II. Experimental methods and corrections

1. *General*

In the present investigation Ilford G5 plates, 7.5 cm × 7.5 cm exposed on the 15th of October 1950 in the United States at a geomagnetic latitude of 64°–65° have been used. The time spent at an altitude of more than 29 km above sea-level was 4.0 hours and most of this time was spent between 29 km and 30 km.

Certain areas of the plates have been scanned for electron-pairs. Two samples of pairs have been selected according to the projected length L of the shortest track. Sample I contains 56 pairs with $L > 1$ mm, found in one of the scanned areas and sample II consists of all found pairs with $L > 3$ mm. Pairs, which go near and nearly parallel to a minimum track, have been rejected as they can have been created by Bremsstrahlung from the particle, which gave rise to the minimum track.

The energy of the electrons has been determined from the multiple scattering. Fowler's coordinate method (10) has been used when determining the mean scattering sagitta $\bar{D}^*$, and large scattering sagittas have been eliminated without replacement until none was greater than 4 $\bar{D}^*$. The measurements have been made with a type M 40000 microscope, manufactured by Cooke, Throughton & Simms and equipped with an eyepiece filar micrometer.

2. *Distortion*

The plates were not distortion-free, but because most tracks had only a small angle of dip, distortion was in most cases negligible. Those tracks, which have been measured with a cell length $t \geqslant 400\ \mu$ have been corrected for C-shaped distortion in the following manner, though this correction has very small influence on the energy spectra.

Let D_i^* be the measured sagitta. D_i^* has an approximately Gaussian distribution with the mean value 0 and a standard deviation which we label σ. Assuming a Gaussian distribution we can write:

$$\langle D_i^* \rangle = 0$$

$$\langle D_i^{*2} \rangle = \sigma^2$$

$$\langle |D_i^*| \rangle = D^* = \sqrt{\frac{2}{\pi}}\, \sigma$$

If every D_i^* is corrected so that $\frac{1}{n} \sum_{i=1}^{n} D_i^* = 0$ after the correction, the correction will be too large because this quantity has a Gaussian distribution $(0, \sigma/\sqrt{n})$. The variable

$$t = \sqrt{\frac{n-1}{n}}\, \frac{\sum_{i=1}^{n} D_i^*}{\sqrt{\sum_{i=1}^{n} D_i^{*2}}}$$

is distributed according to Student's distribution with $n-1$ degrees of freedom, which distribution is asymptotically normal (0, 1). Because the distributions of $\sqrt{\Sigma D_i^{*2}}$ and $\Sigma |D_i^*|$ are similar in shape and because

$$\langle \sum_{i=1}^{n} |D_i^*| \rangle = \sum_{i=1}^{n} \langle |D_i^*| \rangle = n \sqrt{\frac{2}{\pi}}\, \sigma = \sqrt{\frac{2\,n}{\pi}}\, \sqrt{n\,\sigma^2} = \sqrt{\frac{2\,n}{\pi}}\, \sqrt{\langle \sum D_i^{*2} \rangle}$$

it is to be expected that

$$u = \frac{\sum_{i=1}^{n} D_i^*}{\sum_{i=1}^{n} |D_i^*|} \sqrt{\frac{2\,(n-1)}{\pi}}$$

has a distribution which is asymptotically normal (0, 1) approximately. If we have N tracks and not too small values of n, we thus expect

$$S^2(u) = \frac{1}{N} \sum_{\nu=1}^{N} u_\nu^2 = 1$$

34 tracks have been measured with $t \geqslant 400\ \mu$ and the value of $S^2(u)$ was 1.59. For the 24 tracks which had $u^2 \leqslant 2.6$ the value of $S^2(u)$ was 0.88. For the resting 10 tracks the value of ΣD_i was lowered according to the method described by Fowler (10) until u was about 1.3. After this correction the value of $S^2(u)$ was 0.98 for all 34 tracks.

3. *Noise*

To begin with the noise was eliminated by means of measurements with two different cell lengths. In the elimination account was taken of the variation with the cell length of both the noise and the scattering constant.

Notations:

p = momentum of the electron in MeV/c

β = velocity factor of the electron

$\bar{D}^* = \frac{1}{n}\sum_{i=1}^{n} |D_i^*|$ = measured mean sagitta

$D^* = \langle \bar{D}^* \rangle = \langle |D_i^*| \rangle$ = expectation value of the measured sagitta

D = expectation value of the true scattering sagitta

B = expectation value of the noise scattering sagitta

t = cell length in units of 100 μ

$\alpha = \frac{1.8}{\pi}\frac{D}{t}$ = expectation value of the true scattering angle (D is measured in μ).

$p\beta$ can be determined from the well known relation

$$p\beta = k\frac{t^{1/2}}{\alpha} \tag{1}$$

where k is the scattering constant. B and k are assumed to depend on t in the following ways:

$$B = B_0 t^p \qquad k = k_0 t^q$$

From (1) we obtain $D = D_0 t^{3/2+q}$.

If we assume a Gaussian distribution for D_i and B_i, we obtain the relation

$$D^2 = D^{*2} - B^2 \tag{2}$$

For two measurements with cell lengths t_1 and t_2 resp. we then find

$$D_0 = \sqrt{\frac{D_1^{*2} - D_2^{*2}\left(\frac{t_1}{t_2}\right)^{2p}}{t_1^{3+2q} - t_2^{3+2q}\left(\frac{t_1}{t_2}\right)^{2p}}}. \tag{3}$$

An estimator $\bar{D}_0$ for D_0 is obtained from (3) if D^* is replaced by $\bar{D}^*$. Then

$$\bar{B}_1 = \sqrt{\bar{D}_1^{*2} - D_0^2 t_1^{3+2q}} \tag{4}$$

is useful as an estimator for the noise in the small cell t_1. If the noise is known to be B', one can use the convenient estimator

$$D'^2 = \bar{D}^{*2} - B'^2 \tag{5}$$

for the determination of D. The statistical errors and the optimum conditions have been discussed recently by Ekspong (11).

The values of the scattering constant k given by Voyvodic in reference 12 have been used. The total energy E of the electron is related to $p\beta$ in the following way:

$$E = p\beta(1+d), \qquad d = \frac{1}{2}\left(\sqrt{1+\left(\frac{2m}{p\beta}\right)^2} - 1\right)$$

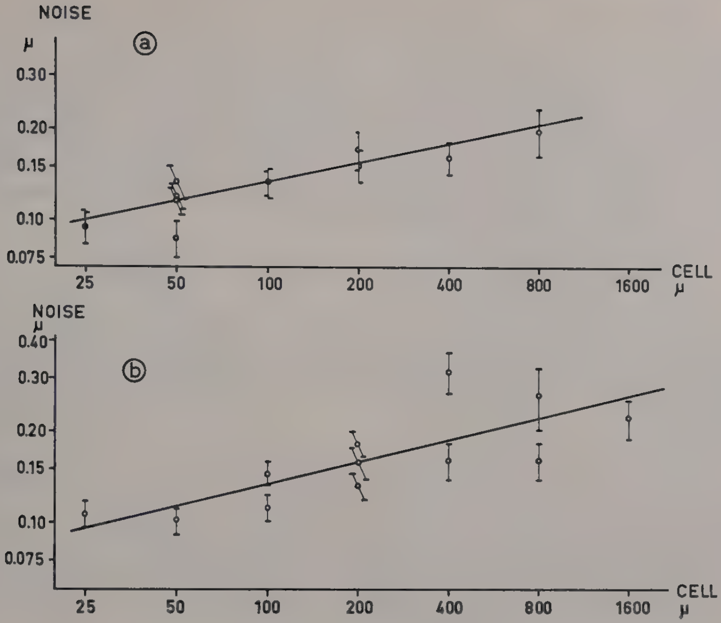

Fig. 3. The result of the two noise-determinations mentioned in the text. The errors indicated are statistical standard errors.

where m is the rest energy of the electron (0.51 MeV). The term d which is approximately equal to $\left(\frac{m}{p\beta}\right)^2$ when this quantity is much smaller than one, could be neglected in all cases, because it is very small at high energies; for instance $d = 0.0026$ when $p\beta = 10$ MeV/c.

Measurements on a very high energy electron gave a preliminary value B' of the noise B. The cells t_1 and t_2 were then chosen in such a way that $\lambda' = D'/B'$ satisfied the following conditions:

$$\lambda' \leqslant 1 \text{ for the small cell } t_1$$

$$2 < \lambda' \leqslant 5 \text{ for the big cell } t_2$$

The values of p obtained from the preliminary noise determination was used when estimating D_0 by means of the relation (3). The noise values obtained by means of (4) showed, however, no larger variations than might be expected statistically. Therefore such values of $\bar{B}$ for which

$$0.15 < \bar{\lambda} < 1.3 \qquad \left(\bar{\lambda} = \frac{\bar{D}}{\bar{B}}\right)$$

were selected for a determination of the noise. The result is shown in fig. 3a. A weighted least square fit of a straight line to the points in the log-log-plot yields

$$B_0 = 0.1327 \pm 0.0008\ \mu \text{ and } p = 0.216 \pm 0.008.$$

The errors indicated include only statistical errors. Care has been taken to keep the systematic and random errors as small as possible (11). During the course of the investigation the construction of the microscope was changed somewhat. Fig. 3b shows the result of a determination of the noise after this operation. In this case one series of values for cells between 200 μ and 1600 μ was obtained from measurements on a straight line drawn on a glass-plate[1] in view of the fact that such a line gives no grain noise. The curvature of the line and the microscope stage did not need to be taken into account. A least square fit yielded

$$B_0 = 0.1342 \pm 0.0011\ \mu \text{ and } p = 0.244 \pm 0.006.$$

For most of the tracks D has been estimated by means of the very convenient formula (5) using the noise values obtained in the determinations mentioned above. A signal to noise ratio $\lambda' = 3$ was aimed at and $1.9 < \lambda' < 6$ was accepted.

4. *Energy loss*

The relatively large length of many of the tracks makes it necessary to take into account energy loss by Bremsstrahlung and ionization. Let us assume that measurements have been undertaken between points at the distance x_1 and x_2 (in mm) from the beginning of the track ($x_2 > x_1$). The correction to be added because of the ionization energy loss is according to Fowler (10):

$$\Delta_j = 0.59\, d \text{ MeV}$$

where

$$d = x_1 + \frac{x_2 - x_1}{2}.$$

If 40 % or more of the energy of the electron seemed to get lost by Bremsstrahlung, the low energy part of the track was excluded. Therefore the quantity to be added as a correction for Bremsstrahlung was chosen as

$$\Delta = p\beta \frac{d}{2\, R\,(p\beta)}$$

where $R(E)$ is a variable radiation length defined by the relation

$$\frac{dE}{dx} = \frac{E}{R}$$

where E is the energy of the electron and $\frac{dE}{dx}$ is the average energy loss per mm path length due to Bremsstrahlung. The correction has been chosen so that it is best for $E = 100$ MeV but the error for other occurring energies is not serious. The value $R(\infty) = 30$ mm has been used (13), and the variation with energy of R according to quantum electrodynamics (14) has been taken into account.

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[1] The author wishes to thank Mr. L. Lundin at the Nobel Institute in Stockholm for supplying high-quality grating-lines.

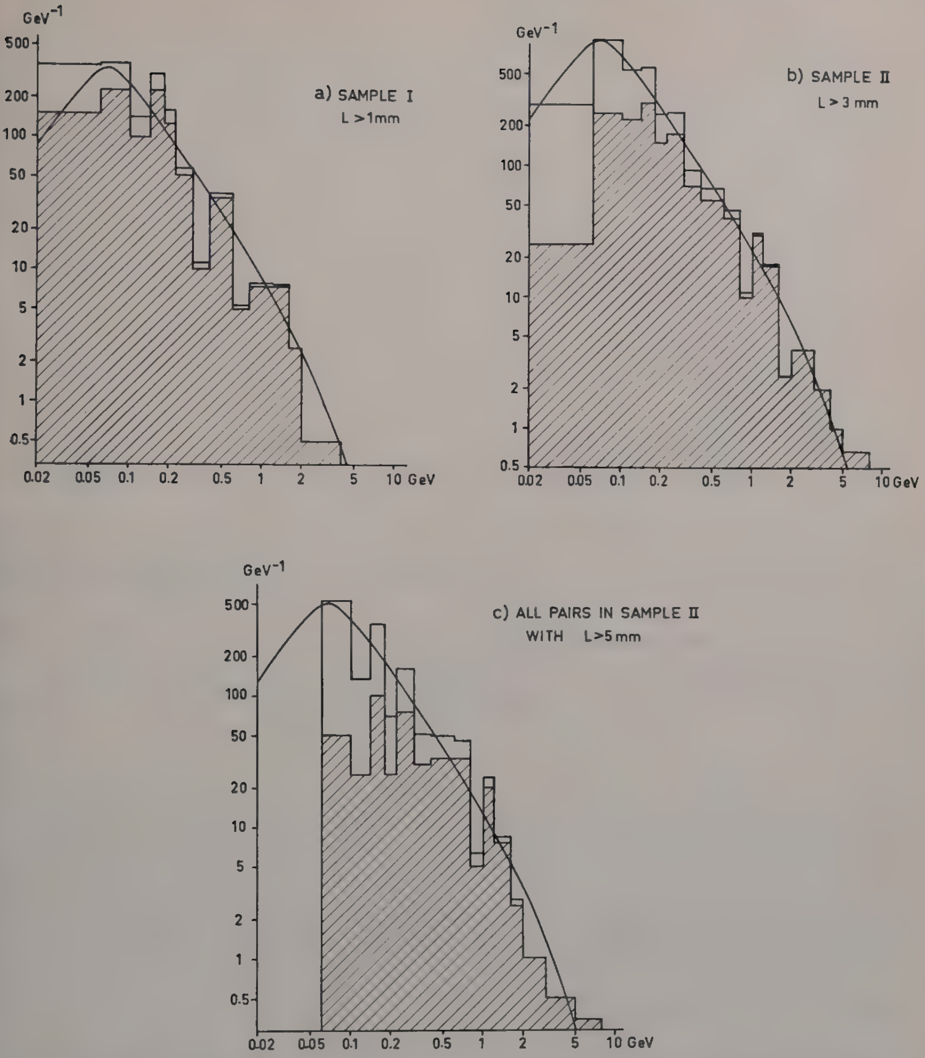

Fig. 4. The smaller values in the histograms show the number of pairs per GeV. The greater values show the photon-spectrum obtained with outscattering and materialization probability taken into account. The smooth curve is the photon-spectrum obtained in section III.

5. *Outscattering*

The pair spectra obtained for $L > 1$ mm (sample I), $L > 3$ mm (sample II) and $L > 5$ mm (selected from sample II) are shown in fig. 4. They show such a large difference in shape that there must be some systematic error. The reason for the variation in shape must be that the probability for outscattering is much greater at low energy than at high energy. The probability $F(E, x)$ that an electron, found to

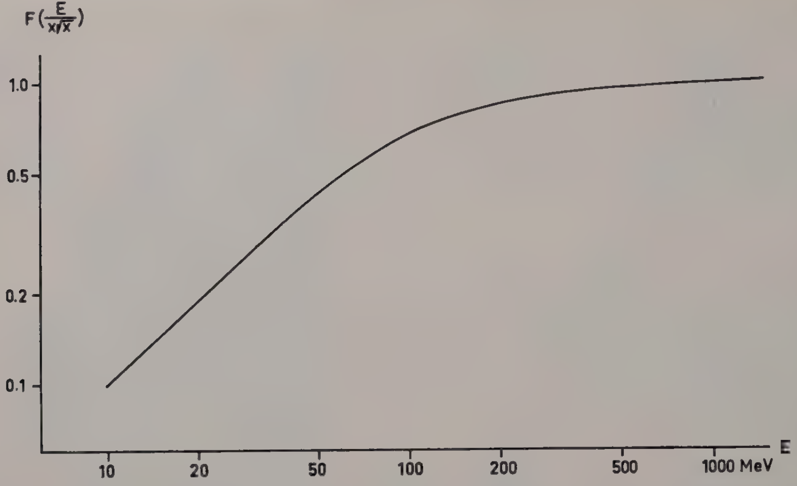

Fig. 5. The probability $F\left(\frac{E}{x\sqrt{x}}\right)$ that the track of an electron of energy E found to start parallel to the emulsion (0.4 mm thick) attains a projected length of x or more, calculated for $x = 5$ mm.

start its path in a plane parallel to the emulsion with the energy E, will travel a projected distance of x or more before being scattered out, has been calculated from the theory of Fermi (15) under the simplifying assumptions mentioned in the appendix. The result is shown in fig. 5. It is proved in the appendix that $F(E, x) = F\left(\frac{E}{x\sqrt{x}}\right)$ so that the calculations need be performed only for one value of x. If pairs, for which the projected length is longer than L for both tracks, are selected, the intensity is considered to be reduced by outscattering with a factor $F\left(\frac{E}{2L\sqrt{L}}\right)^2$ where E is the energy of the pair. As pointed out in the appendix, the real reduction is certainly somewhat greater for small energies for which F^2 is appreciably smaller than one.

6. *Materialization probability*

The fraction of the photons of energy E in a homogeneous beam that will materialize in a small volume dV when passing through it is proportional to $\sigma(E)\, dV$, where $\sigma(E)$ is the cross-section for pair creation. The photon spectrum has been obtained from the corresponding pair spectrum by applying to each pair with the energy E the statistical weight

$$\frac{\sigma(\infty)}{F\left(\frac{E}{2L\sqrt{L}}\right)^2 \sigma(E)}.$$

The variation of $\frac{\sigma(E)}{\sigma(\infty)}$ with energy has been obtained from quantum electrodynamics (14).

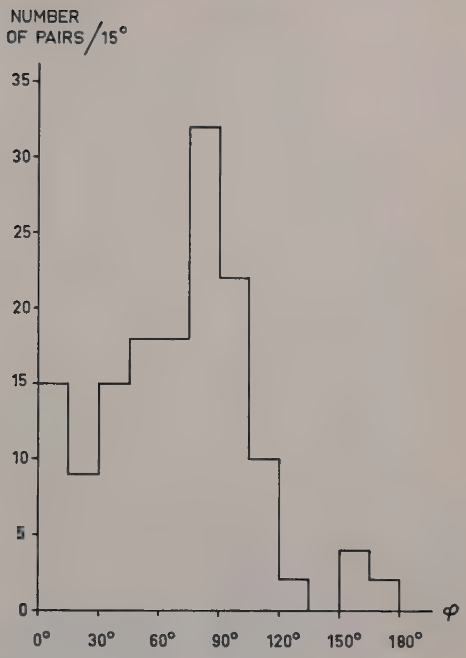

Fig. 6. The distribution in projected zenith-angle ϕ of all measured pairs.

III. Experimental results

1. The angular distribution

The best way to analyze the result would no doubt have been according to the method given in section *A*. The statistical material does not, however, seem to be large enough to justify the tremendous numerical work necessary when using that method. Therefore a more conventional discussion of the results has been undertaken.

The distribution in projected zenith angle ϕ of the pairs measured is shown in fig. 6. This angle does not differ seriously from the true zenith angle θ_γ. The solid angle $d\,\omega_\gamma$ corresponding to an angular interval $d\,\phi$ is independent of ϕ. The angular distribution differs markedly from that obtained by Carlson *et al.* (1). The latter was isotropic for $\phi < 90°$ and fell off rapidly to zero for $\phi > 90°$. The present distribution shows a strongly pronounced peak at about 90°. This peak would be still more marked for the distribution of the number of pairs per steradian as a function of θ_γ because θ_γ generally has a value somewhat more near 90° than ϕ has. The difference between the two experimental distributions is certainly due to the fact that the Bristol plates were exposed at a greater atmospheric depth (21 km above sea-level) than the plates used in the present investigation. In the latter case the amount of matter in which meson-producing collisions can occur is much greater in the direction near 90° than for smaller zenith angles. The small central peak is believed to be due to another stack placed some distance above the one used in the present investigation during the flight. Fig. 7 shows the distribution in ϕ for all scanned pairs. It is similar to that in fig. 6. Because most of the pairs in fig. 7 are shorter than 1 mm the difference between ϕ and θ_γ is generally greater for this distribution than for that in fig. 6.

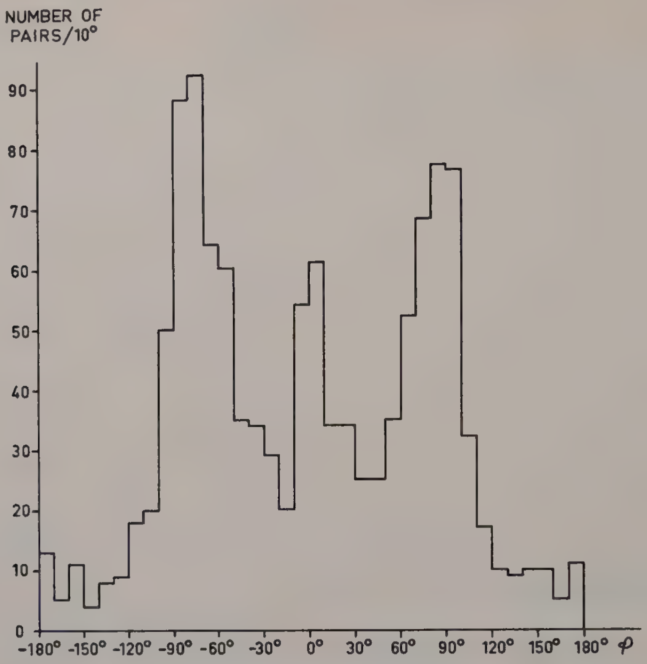

Fig. 7. The distribution in projected zenith-angle ϕ of all scanned pairs.

2. *The energy distribution*

A photon with the energy E can arise in the decay of π^0-mesons of energy between $\varepsilon(E)$ and ∞, where

$$\varepsilon(E) = E + \frac{\varepsilon_0^2}{4E} \tag{6}$$

ε_0 is the π^0-meson rest energy. The angle ϑ between the direction of the π^0-meson and the photon depends on E and ε according to

$$\cos\vartheta = \frac{\varepsilon - \frac{\varepsilon_0^2}{2E}}{\sqrt{\varepsilon^2 - \varepsilon_0^2}} \tag{7}$$

and varies from $\vartheta = 0$ when the π^0-meson energy ε has the value $\varepsilon(E)$ to $\vartheta_{\max} = \text{arc-}\cos\beta$ when $\varepsilon = 2E$ and then back to $\vartheta = 0$ as $\varepsilon \to \infty$. βc is the velocity of the π^0-meson. A plot of $\vartheta_{\max}$ as a function of E is shown in fig. 8. From this we see that a photon with the energy 0.26 GeV or more can form an angle of at most 15° with its parent π^0-meson. Assuming that the primary cosmic radiation has an isotropic angular distribution in space outside the region where the earth's influence is important, it seems reasonable to assume that the π^0-mesons are isotropically distributed for zenith angles smaller than 90°. Then the photons with energy > 0.26 GeV and zenith angle $< 75°$ have all their parent π^0-mesons within this isotropic distribution and if cascade

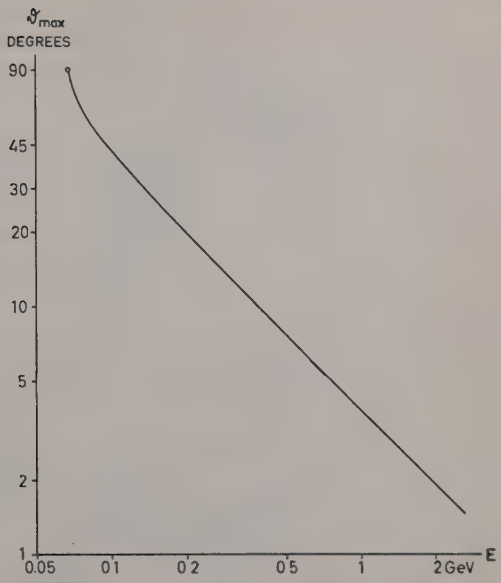

Fig. 8. The maximum angle $\vartheta_{\max}$ between the direction of a photon of energy E and its parent π^0-meson.

processes are neglected the differential energy-spectrum of the π^0-mesons $F(E)$ may be obtained from that of the photons $N(E)$ from the relation

$$F\left(E+\frac{\varepsilon_0^2}{4E}\right)=\frac{1}{k}\left|\frac{dN(E)}{dE}\right| \tag{8}$$

where k is a constant. (Section A)

Fig. 9 shows the photon spectra obtained from pairs in the projected zenith angle intervals (0°, 75°), (75°, 105°) and (0°, 180°). A smooth curve has been fitted to the (0°, 180°)-spectrum in the following way. The differential photon spectrum was assumed to have the form

$$N(E)=A\cdot E^{-p}$$

where E is the photon energy in GeV. Two least square fits of straight lines to the experimental points in the log-log-plot gave the following result:

$0.26 \leqslant E \leqslant 1.4$	$A=29.7 \pm 1.4$	$p=1.53 \pm 0.06$
$1.4 \leqslant E \leqslant 6.5$	$A=36.4 \pm 7.4$	$p=2.22 \pm 0.21$

The total energy represented by all pairs of energy >3 GeV was 37 ± 16 GeV. The statistical error includes the error due to the expected statistical fluctuation of the number of pairs according to Poisson's statistics. If the same exponent p is assumed between 3 GeV and ∞ and the value from the second least square fit is used for $N(3\ \text{GeV})$, we obtain

$$p=2.8 \pm 0.4 \quad \text{for} \quad E>3\ \text{GeV}.$$

This result is represented by the dotted line in fig. 9c. To the statistical standard error given there contribute in the last case systematic errors, depending on distortion,

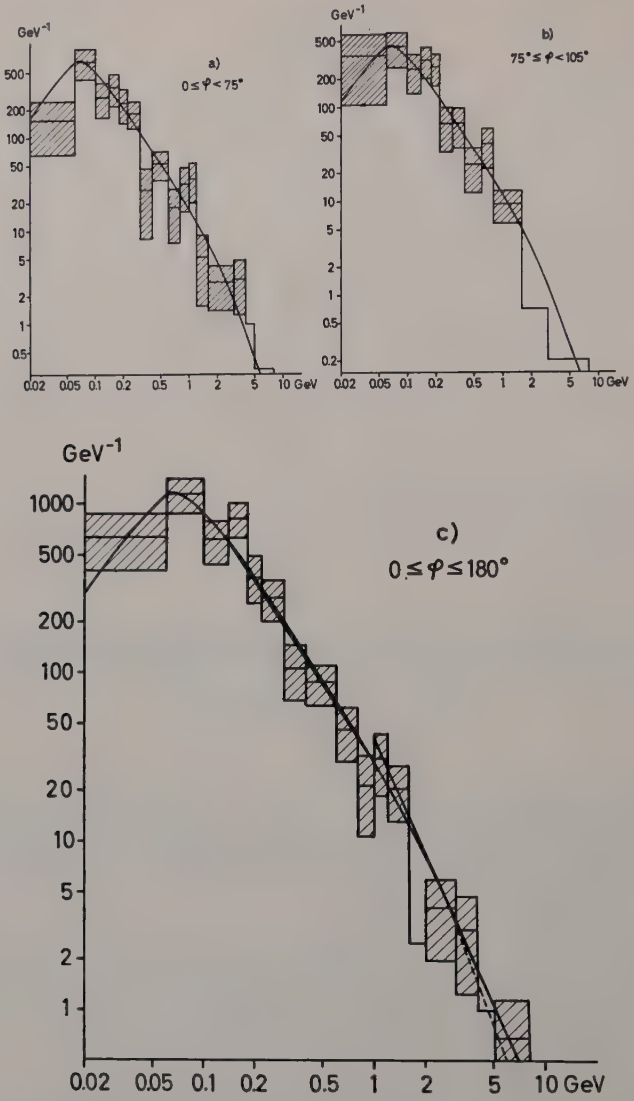

Fig. 9. Energy-spectra for photons in the projected zenith-angle intervals indicated in the diagrams. The statistical standard errors are indicated, when the number of pairs in the energy-interval is greater than one. The straight lines and the smooth curve in diagram c) are explained in the text. The smooth curve in diagrams a) and b) is the same as that in diagram c).

spurious scattering and scanning efficiency, which are difficult to estimate objectively. Therefore the good agreement with earlier results must not be stressed too much in this case. After these lines had been drawn, a smooth curve was drawn in accession to them. The curve for $E < \frac{\varepsilon_0}{2}$ was obtained from the fact that there

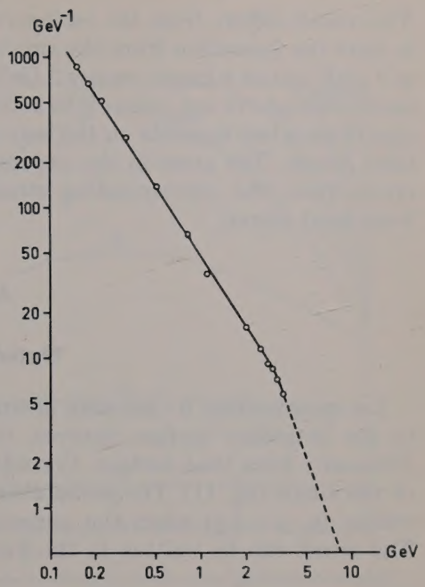

Fig. 10. The differential π^0-meson production spectrum obtained from the photon-spectrum (the smooth curve in fig. 7c). The dots are the results of computations according to formula (8) in section B.

should be mirror symmetry in the log-log-plot about the line $E = \frac{\varepsilon_0}{2}$ if the theory of Carlson *et al.* (1) is applicable. In this investigation it has not been possible to obtain a value of the π^0-meson mass, because the choice of $L \geqslant 3$ mm for most pairs makes the outscattering so large for the low energy part of the spectrum that its form cannot be determined experimentally.

As can be seen from fig. 9 it is possible to fit the curve obtained for the (0°, 180°) spectrum to the two other spectra. The spectra for (0°, 75°) and (75°, 105°) do not differ appreciably from each other. Cascade processes in the atmosphere are negligible in the former interval, but may be important in the second. Because the distance from the edge to the center of a plate is about one radiation length cascade processes in the plates are never negligible, but they are considered to give the same contribution for all directions. Since cascade processes may thus have played a greater part in the (75°, 105°) interval than in the (0°, 75°) interval it seems justified to conclude that they are not decisive for the form of the spectrum. This may be due to the fact that in fairly large energy intervals the photon energy spectrum obeys a power law with a constant exponent p. If the exponent had been the same up to infinity it would not have changed during the propagation of the photons through matter according to shower theory. In the present case the energy intervals, where the exponent may be considered to be constant, are probably large enough for the exponent not to change seriously during the one or two radiation lengths that are possible.

In view of the above discussion it is probably permissible to use (8) when deducing the π^0-meson production spectrum from the photon spectrum. The result is shown in fig. 10. The exponents obtained are

$$0.25 \text{ GeV} \leqslant \varepsilon \leqslant 3 \text{ GeV} \qquad p = 1.53 \pm 0.2$$

$$\varepsilon \geqslant 3 \text{ GeV} \qquad p = 2.8 \pm 0.4$$

This result differs from the earlier ones, which have been put together by Kim (16) in that the transition from the smaller exponent 1.5 to the larger 2.8 does not occur at 1 GeV but at a larger energy 3 GeV, or perhaps even higher if the systematic errors mentioned above are taken into account. The abrupt transition has perhaps no real significance but depends on the way in which the curve for the photon spectrum has been drawn. The error in the exponent p seems difficult to estimate. The statistical errors from the corresponding straight lines fitted to the photon spectrum have been used above.

APPENDIX

Outscattering theory

Let us introduce a Cartesian coordinate-system (x, y, v) with the xv-plane parallel to the boundary surface between the emulsion and the glass and situated at the distance z from that surface. Consider a track beginning at $(0, 0, 0)$ in the direction of the x-axis (fig. 11). The probability $S(E, x, y)\,dy$ that the track has its y-coordinate within $(y, y+dy)$ when the x-coordinate is x has been calculated by Fermi (14). The result can be written in the form

$$S\left(\frac{E}{x\sqrt{x}}, y\right) = \frac{e_0}{\sqrt{2\pi}\,x\sqrt{x}}\, e^{-\frac{e_0^2 y^2}{2x^3}} \tag{1}$$

where $e_0 = \frac{E}{1.5}$ when the scattering constant is taken to be $k = 30$ (grad · MeV · $(100\,\mu)^{-1/2}$), the energy E of the electron is measured in MeV and x and y are measured in mm. The substitution

$$W = W\left(\frac{E}{x\sqrt{x}}, y\right) = \frac{e_0 y}{x\sqrt{x}} \tag{2}$$

yields

$$S\left(\frac{E}{x\sqrt{x}}, y\right) dy = f(W)\,dW$$

where

$$f(W) = \frac{1}{\sqrt{2\pi}}\, e^{-\frac{W^2}{2}}$$

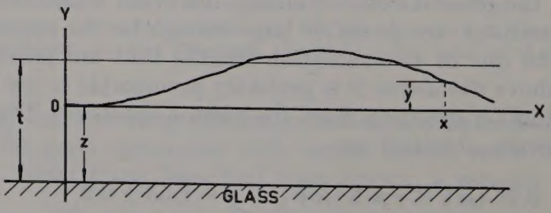

Fig. 11. Illustration to the outscattering theory.

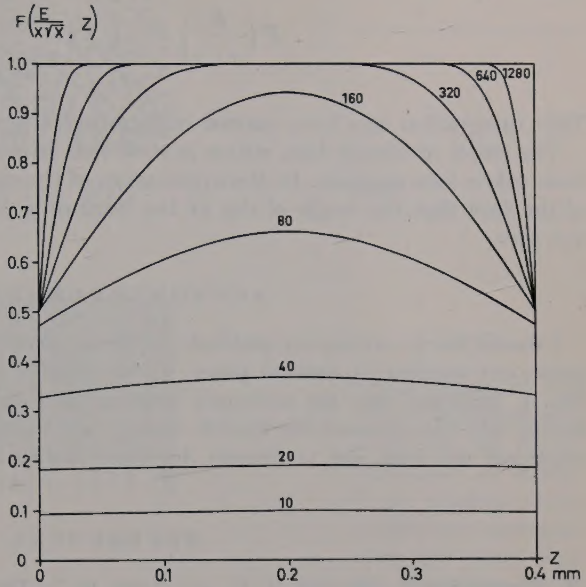

Fig. 12. The probability $F\left(\frac{E}{x\sqrt{x}}, z\right)$ that an electron starting at the distance z from the glass and parallel to it attains a projected length of x or more in an emulsion 0.4 mm thick. The numbers attached to the curves are the electron energies E in MeV, when $x = 5$ mm.

The probability, $F(E, x, z)$ that the electron is not scattered out, until it has passed the x-coordinate x is

$$F\left(\frac{E}{x\sqrt{x}}, z\right) = \int_{y_1}^{y_2} S\left(\frac{E}{x\sqrt{x}}, y\right) dy$$

where $y_1 = -z$, $y_2 = t - z$ (t = the emulsion thickness). An approximation which involves an underestimation of the outscattering has been made because a track like that in fig. 11 is considered not to be scattered out. However, the error introduced in this way does not seem to be very large because the electron is rarely scattered back into a layer which it has already left (13). The substitution (2) yields

$$F\left(\frac{E}{x\sqrt{x}}, z\right) = \int_{W_1}^{W_2} f(W)\, dW$$

where $$W_i = W\left(\frac{E}{x\sqrt{x}}, y_i\right) \qquad i = 1, 2.$$

$F\left(\frac{E}{x\sqrt{x}}, z\right)$ has been calculated for $t = 0.4$ mm and $x = 5$ mm. The result is shown in fig. 12.

Finally let us consider the case when all possible values of z are equally probable. The probability that the electron will reach the x-coordinate x without being scattered out is

$$F\left(\frac{E}{x\sqrt{x}}\right)=\frac{1}{t}\int_0^t F\left(\frac{E}{x\sqrt{x}},z\right)dz$$

This integration has been carried out numerically and the result is shown in fig. 5.

The effect of energy loss, which is such that it increases the outscattering, has not been taken into account. In the application of the result, account has not been taken of the fact that the angle of dip at the beginning of the electron tracks is generally not zero.

ACKNOWLEDGEMENTS

I should like to express my gratitude to Docent Gösta Ekspong, who suggested the investigation and supplied the scanned plates, for his valuable advice and ideas. I also wish to thank Mrs. I. Backlund, who has performed some of the numerical calculations, and Professor M. Melvin, who has corrected the English. Finally, my thanks to Professor K. Siegbahn, who has supervised the work, and to *Svenska Atomkommittén* for financial support.

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Tryckt den 28 mars 1958

Uppsala 1958. Almqvist & Wiksells Boktryckeri AB